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Interpretation of Data Showing Something has One Effect Sometimes and a Different Effect in other Circumstances: Theories of Interaction of Factors

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Abstract
A possible explanation of interaction is that quantities derived from the independent variables separately add together, but then a curvilinear relationship intervenes between their total and the dependent variable observed. It is shown that two different theories of this type are always available to explain crossover interaction in a 2x2 table. For example, one theory may say that a good outcome occurs when there is an approximate match between values associated with the independent variables, and the other theory that a good outcome occurs when the total of values associated with the independent variables is either decisively small or large, with poorer outcome resulting from intermediate values.

Introduction
Factorial experimentation involves manipulating two (or more) x's, and observing y at all combinations of the values of x_1 and x_2. If the effect on y of x_1 depends upon what x_2 is, there is said to be an interaction between the x's.

A possible explanation of interaction is that (a) quantities derived from the independent variables separately add together, but (b) a curvilinear relationship intervenes between their total and the dependent variable observed (see Hutchinson, 2004). This idea is not new. Indeed, once it comes to mind, it is a very obvious one. The reason it is worth publicising is that it seems not to be widely taught and not to come to researchers' minds spontaneously.

Interaction was found, for example, in computer simulation of logistics operations by McGee et al. (2005). Consider a logistics system that supports operations, and the effects of characteristics of the system on the operational availability of equipment. Capability to use express service for shipments of parts and capability to repair parts locally will both be good, but to some extent they are substitutes for each other, and having both will not be much of an improvement on having one of them.

A common pattern in reports of research is for interaction to be found to be statistically significant, for the researchers to make a song and dance about the novelty and importance of this, but then for no substantive explanation or interpretation to be given.

The next Section discusses the results of McGee et al. (2005). The explanation that will be proposed is one of positive but decreasing returns for effort (i.e., the slope of the curvilinear relationship is positive but decreasing). Attention then turns to curvilinearity that actually reverses in direction, and leads to crossover interaction. Not merely one, but two simple and attractive explanations for interaction can always be found, if there are only two factors and only two categories of each. They may be relevant to, for example, the idea that that the tone of a message needs to match the personality of the audience receiving it, if attitude or behaviour change is to occur. Finally, there is a short discussion Section.

A Logistics Simulation Example

McGee et al. (2005) had several factors in their study rather than two. It turns out that the specific pattern of the interactions gives some evidence for a theory in which things add together but then there is curvilinear dependence on the total. The results in McGee et al. indicate that all of the following four factors are good for the measure of performance, operational availability:

(A) Waiting until there is a truckload of parts is not necessary,
(D) Shipments may be made by express service,
(G) Equipment is reliable,
(J) There is capability to repair parts locally.

(The capital letters are identifiers as in McGee et al., but in the case of statements (A) and (D), positive coding in McGee et al. corresponded to the negation of the statement.)

All of those results of factors considered singly are in accordance with common sense. What is of interest is that there are interactions of (D) with each of (A), (G), and (J), and that these interactions are such that (D) being true is less important if (A) is true, if (G) is true, and if (J) is true. This rather suggests that positive quantities derived from (A), (D), (G), and (J) being true combine additively, with operational availability improving less than linearly with the sum. (This would imply that the other two-way interactions between these variables do exist, even though they were not large enough to be reported in McGee et al.)

The present paper is concerned with understanding a phenomenon, interaction, that may appear even in 2x2 tables. For simulation experiments, an enormous design space may be of interest --- there is a case study in Kleijnen et al., 2005, for which the starting point was 40 factors each at 40 levels --- and it should be conceded that the priorities for data processing may be utterly different. Note also that when interaction is only quantitative, rather than crossover, there is a choice over how seriously to take it, as it may be possible to find some reasonably simple and meaningful transformation of the dependent variable such that there is no longer any interaction.

McGee et al. were concerned with logistics in a military context. Other examples of interaction may be found in actual military operations. From a defender’s point of view, it is bad to make mistakes, and it is bad if the ground attacker has good technology --- but the effect of a technology may be much greater when the defender makes a certain type of mistake, leading to utterly one-sided combat. Examples of this include the combination of air superiority and failure to detect a ground attack (being surprised is a disaster when the defenders are sheltering rather than manning their vehicles), and the combination of advanced sights and inadequate concealment of targets (advanced sights that penetrate darkness and sandstorm are only useful if the defenders fail to hide their vehicles behind a hill). Interactions between different errors by the defenders and between defenders’ errors and attackers’ technology were examined by Biddle (1996).

Argument for 3 x 3 Experiments, Rather than 2 x 2

Let the categories of x_1 be A and B, the categories of x_2 be C and D, and the observations of the dependent variable y be as follows.

	C	D
A	1	4
B	3	2

There is “crossover” interaction: moving from C to D increases the response in condition A, but decreases it in condition B. Further, moving from A to B increases the response in condition C, but decreases it in condition D.

Suppose we are lucky enough to have a theory that specifies what it is about x_1 and x_2 that is adding together and determining the level of response, and roughly how much of it is associated with A, B, C, and D. Without loss of generality, these amounts can be taken as 0, 1, 0, and 2, respectively.

- First, let us add these together, and let the result be a total t.

	C (0)	D (2)
A (0)	0	2
B (1)	1	3

The totals t, in order from 0 to 3, are shown below along with the corresponding y's.

t:	0	1	2	3
y:	1	3	4	2

Thus a theory in which the quantities add together, and then y is an inverted-U shaped function of the result (i.e., it first increases, then decreases), will explain the dataset.

- Second, let us subtract the quantities, and call the result a difference d.

	C (0)	D (2)
A (0)	0	2
B (1)	-1	1

The differences d, in order from -1 to 2, are shown below along with the corresponding y's.

d:	-1	0	1	2
y:	3	1	2	4

Thus a theory in which the quantities are subtracted, and then y is a U-shaped function of the result (i.e., it first decreases, then increases), will explain the dataset.

Thus there will always be two different theories available to explain a 2x2 table. (Even more theories will be available if there are no preconceptions about the quantities associated with x_1 and x_2.) In the case where small values of y are better than high values, the first theory says that a good outcome occurs when the total t is either decisively small or decisively large, with poorer outcome resulting from intermediate values; and the second theory says that a good outcome occurs when there is an approximate match between the values associated with x_1 and x_2. Hutchinson (2008) discussed this in the context of the dependence of house prices on characteristics of the house and characteristics of its location. The two competing theories there were that for a house to be highly valued, (a) it and its location should in total be either highly urban or highly suburban, not in between, or (b) there should be a match between the characteristics (in an urban vs. suburban sense) of the house and its location.

Are these really different theories, or is one somehow a disguised version of the other? How can a decision be made between them? Yes, they are different, as can be seen by considering the result when a category intermediate between A and B is paired with a category intermediate between C and D.

- The sum of two intermediate quantities is intermediate, neither decisively small nor decisively large, so the first theory predicts the outcome will be poor.

- There is an approximate match between the category of x_1 and the category of x_2, so the second theory predicts the outcome will be good.

Consequently, a 3x3 table of results will enable us to decide between the two theories.

Example Concerning Compatibility Between a Message and Its Audience

Attempts to change attitudes and behaviour have often had disappointing results. Yet great changes in public sentiment have occurred in regard to some issues in recent decades (e.g., smoking is less tolerated). An idea that has been proposed to explain this variation is that the tone of a message needs to match the personality of the audience receiving it. If the necessity of matching message to audience is a reality, it refers to crossover interaction: one thing is superior to another in condition 1, but is inferior in condition 2.

Goldstein (1959) found that a strong fear appeal receives greater acceptance among those he referred to as copers than among those he referred to as avoiders, while a minimal fear appeal receives greater acceptance among avoiders than among copers. He was able to refer to other literature supporting the idea of individual differences in reactions. There has been much subsequent research. According to a review by Atkin (2001, p. 23), "Effectiveness can be increased if message content, form, and style are tailored to the predispositions and abilities of the distinct subgroups". Later in that review (pp. 31-32), there is discussion of mechanisms causing health campaigns to fail. These mechanisms will apply to some audiences and in some circumstances, while for other audiences and in other circumstances, the campaign would have its intended effect. Evidently, then, the hypothesis is that what matters is the difference between some aspect of presentational style (e.g., how graphic and threatening it is) and some aspect of the people receiving the message (e.g., the extent to which they are sensation seekers), with effectiveness declining either side of some optimum. Jones and Owen (2006) draw attention to the variety of different findings concerning the effect of level of threat on likelihood of behavioural change, including the possibility of an inverted-U relationship.

The implication of the present paper is that if it is credible that maximum change in attitude or behaviour occurs when the difference between excitement (for example) of the message and of the audience is small, it will also be possible to invent a theory saying that excitement as a characteristic of the message and excitement as a characteristic of the audience add together, and maximum change occurs when the total is either small or big and is smaller in between. But perhaps it is unappealling that maximum change occurs when total excitement is either small or big and is smaller in between? The rely to this is that it is difficult to move from an abstract model to an appropriate name, and the problem may lie in the name. Suppose that "excitement" is really a distinction between excitement and rationality. Theorise that maximum change occurs when either total excitement is big or total rationality is big (and is smaller in between) --- the idea that was unappealling now has a certain plausibility.

Discussion

To interpret the results from the multi-factor study of McGee et al., there were several steps.

- Propose a theory, in the sense of identifying that several things are expected to have a positive effect.
- Code the factors in such a way that all their individual effects will be positive.
- After fitting a model with individual effects and interactions, examine whether the interactions are positive or negative.
- When the interactions are found to be negative, conclude that it appears there are decreasing returns for effort.

Having a theory also appears important when there is crossover interaction.

- Faced with the puzzle of opposite effects in different circumstances, even the general idea of a U-shaped or inverted-U shaped dependence is a step forward.

- Two specific proposals are potentially of wide application: small sum or large sum both being good, or small difference being good.
- Then one needs some reason to think that category E (for example) lies between A and B on factor x_1 and that category F (for example) lies between categories C and D on factor x_2: the result for the combination EF will decide between the two proposals.

When an interaction is found, there is naturally a demand for theory to explain the complicated result. It seems fair to conclude that quite simple ideas may help, and may even suggest future lines of research.

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