Simulation of rural travel times to quantify the impact of lower speed limits

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Abstract

The number and severity of rural road crashes has been shown to decrease with reduced travelling speed. One method of reducing the travelling speed on rural roads is to reduce the speed limit of those roads. Despite the considerable road safety benefits resulting from reduced speed limits, public opposition to the change exists. One of the main concerns of the public is the perceived increase in travel times associated with a reduction in speed limit.

This study quantifies this increase in travel time on a rural road if the sign posted speed limit of 110 km/h was replaced with the default speed limit of 100 km/h. A Markov simulation model of travel time on an undivided rural road was developed. The model includes factors such as vehicle speed distributions, vehicles travelling in the opposite direction and the ability for vehicles to perform an overtaking manoeuvre. Real data collected on rural roads in South Australia that have had a similar reduction in speed limit are used to define the distribution of speeds of vehicles before and after the change in speed limit.

The model shows that the increase in travel time is less than is first predicted by considering only the allowed speed limit. The model also shows that a driver who desires to maintain a constant travel speed must overtake more often when the speed limit is higher than when it is lower.

Keywords

Speed Limit, Rural Road, Travel Time, Simulation Model, Overtaking

Introduction

It is clear that reducing the speed of all vehicles on a road network has a positive safety benefit [1,2]. The safety benefits of reduced travel speed include a reduction in the number and severity of crashes that occur on the road. One of the major ways of achieving this is to reduce speed limits [2].

In South Australia, speed limits were reduced on 1060 km of rural roads from 110km/h to 100km/h, resulting in a reduction in casualty crashes of 12% and crash savings of $9.5 million per year [1]. An opportunity existed to use the data from these changes to explore the issue of travel times.

Despite the road safety benefits of reduced speed limits, a reduction in speed limit is often perceived by the public as a change that severely increases the time taken to complete a journey. This perception was shown in a metropolitan setting when considering a reduction in speed limit from 60 km/h to 50km/h [3]. A simple analysis, using equation (1) indicates that the travel time will indeed increase after a reduction in speed limit. For a speed limit change from 110 km/h to 100 km/h, an increase in time of 10% is expected.

$$Time = \frac{Distance}{Speed} \qquad (1)$$

However, other parameters also affect the time taken to travel a road. These other parameters include the influence of other traffic on the road and the length of road which can actually be traversed at the speed limit, as restrictions may be placed on the driver due to geometrical constraints. In order to accurately calculate the increase in time taken due to a reduction in speed limit, these other parameters should also be taken into account.

The first of these additional parameters, the influence of other vehicles on travel time is investigated in this study. At present the influence of road geometry independent of overtaking constraints is not modelled. Other vehicles on the road network have two immediate qualitative effects. Firstly, when a slower vehicle is encountered by a faster vehicle, the faster vehicle must slow to the speed of the slower vehicle to avoid a collision. The number and distribution of slower vehicles on the road network therefore influence the travel time on a rural route. Secondly, overtaking can only be performed when either a dedicated overtaking lane is present, or there is no oncoming traffic and there is sufficient sight distance for the faster vehicle to overtake.

However, the change of speeds of each individual vehicle is not equal to the change in speed limit. This has been observed in both a metropolitan setting [4] and also a rural setting [1]. Furthermore, the reduction in speed is not uniform for all vehicles in the speed distribution [4].

To investigate the effect of the variable slowing of traffic after a speed limit change, a simple mathematical model of travel time was developed using several Markov chains. The mathematical model was developed to provide a tool which would be useful for engaging the public in the debate over a reduction in speed limits. As a tool, the model provides additional information for the public relating to the journey in addition to the increase in travel time that results from a decrease in speed limit. As a simple mathematical model, the absolute values that are reported need to be validated, but the relative values provide insight for the public. Markov chains are frequently used to describe systems where queues are involved, such as calls arriving at a call centre, and customers waiting in a line to be served at a shopping centre. In traffic theory they have been used to describe the level of congestion on links of a highway [5] and for an analysis of travel time on a unidirectional highway [6]. However, to the authors' knowledge, Markov chains have not been used to describe the discrete states of travel which are determined by following of slower vehicles until an overtaking opportunity presents itself on a single lane rural road.

Computational software is available to determine various aspects of travel on rural roads in an Australasian context [7], but typically a detailed input of the distribution of speeds of vehicles on the road is not possible [8,9]. Generally, the computational software that is available are micro-simulations [7] where various vehicles are placed on a length of highway and their interactions with each other are simulated. The model developed in this paper considers only the vehicles which interact with the modelled vehicle: slower vehicles that are in front of the modelled vehicle and all vehicles travelling in the opposite direction to the modelled vehicle.

The Markov chain used in this study allows a driver to maintain a steady speed unless he or she is restricted by a driver in front who has a slower desired speed. The model also allows for passing only when sufficient sight distance is available and there are no opposing vehicles, or when a passing lane is available. After a large number of simulations, the distribution in travel times for a particular road was obtained.

Methods

Data

The travelling speeds of cars on rural roads was taken from the raw data used to compute the summary statistics of average speed presented in Long et al [1]. Data in this study was obtained for sites where a speed limit reduction from 110 km/h to 100 km/h occurred on July 1 2003. One week of speed data was used to derive the speed distribution at each site. For simplicity, two sites at which data were available were used in this study: one near Pt Clinton, and one between Murray Bridge and Jervois. They were chosen because they represented sites with the most traffic, and each had a high proportion of vehicles travelling at a free speed. A free speed for the purpose of this study was defined as a vehicle with headway, or time gap between itself and the previous vehicle, of not less than four seconds. Free speeds are commonly used to represent the chosen speed of a driver independent of other vehicles on the road. Details of the two sites chosen are presented in Table 1.

Table 1: Details of the sites selected for use in this study.

Site	Speed limit	Date range of speed observations extracted	Number of vehicles during observation period	Vehicles at free speed	Average free speed (km/h)
Port Clinton	110	29 Jan – 4 Feb 2003	12 730	84.7%	101.1
	100	29 Oct – 4 Nov 2003	12 459	85.7%	99.3
Murray Bridge - Jervois	110	4 Dec – 10 Dec 2002	8 797	90.3%	96.8
	100	10 Sep – 16 Sep 2003	7 882	90.8%	96.2

The distributions of free speed for each site are shown in Figure 1. This demonstrates that the magnitude of speed reduction following the speed limit change does not simply match the difference in speed limit.
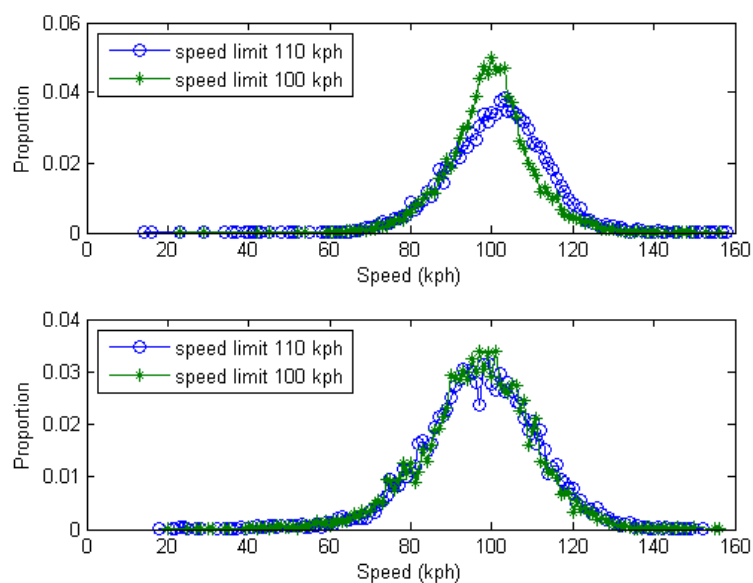

Figure 1: Speed distributions before and after the change in limit for each site: Pt Clinton (above) and Murray Bridge - Jervois (below)

Model

The model uses three variables in order to determine the travelling time and average speed of a vehicle on a rural road. These three variables are:

1. The speed of the modelled vehicle,
2. The status of the lane that is being driven in (explained later)
3. The presence of an opposing vehicle that would restrict an overtaking manoeuvre.

The first variable is used to determine the travel time for each journey. The remaining two variables are used to determine if an overtaking manoeuvre can be performed. Each variable is modelled using a Markov model and are briefly described in the following sections. A more thorough description of the model is included in the Appendix.

A Markov model

Briefly, a Markov model is a memory-less stochastic process. If $X(t)$ is a continuous time Markov chain then a notation to indicate that the Markov chain is in state j at time t is given by equation (2).

$$X(t) = j \tag{2}$$

If we assume that the Markov chain has a finite number of possible states, I, and that the notation $P\{p|q\}$ is the probability of p given q; then the definition of a continuous time markov chain is given in equation (2).

$$P\{X(t + \Delta t) = j | X(t) = i, X(u) = x(u), 0 \le u \le t\} = P\{X(t + \Delta t) = j | X(t) = i\} \tag{3}$$

For all $t, \Delta t \ge 0$ and $i, j, x(u) \in I$.

Or, more simply; the probability of making a transition from a state $X = i$ to state $X = j$ is dependant only on the state i and not on the complete time history of X. That is, a Markov chain is memoryless.

The particular Markov model used is built from three continuous time Markov models:

$$X(t) = \{S(t), L(t), H(t)\} \tag{4}$$

Where
$S(t)$:   speed at time t (km/h)
$L(t)$:   lane state at time t
$H(t)$:   state of the safe distance at time t.

Speed model

The modelled vehicle has a maximum speed and a desired speed equal to the speed limit. The speed of the modelled vehicle may be slower than the desired speed because the modelled vehicle will encounter other vehicles travelling at a speed less than the speed limit. Each time the modelled vehicle encounters a slower moving vehicle, the modelled vehicle duplicates the speed of the slower vehicle until an overtaking opportunity arises. If further successive slower vehicles are encountered while the modelled vehicle is already following a slower vehicle, the modelled vehicle duplicates the speed of the slowest vehicle. This speed is maintained until an overtaking opportunity arises and the modelled vehicle overtakes all vehicles in front of it.

Slower (and faster) vehicles are distributed in front of the modelled vehicle using a Poisson distribution. The Poisson distribution is used in order to keep the model simple. It is the simplest distribution possible for a Markov model and does not model platooning of vehicles and consequently requires the number of overtaking opportunities to be high and the density of traffic low.

Lane model

Each lane model has three different types of lane:

$$L(t) = \begin{cases} 1 & \text{where no overtaking is allowed, eg. on bends} \\ 2 & \text{where overtaking is allowed if safe} \\ 3 & \text{where an overtaking lane exists} \end{cases}$$

Modelling was conducted in two phases. The first phase assumed a fixed length of 100km of road with each of the three lane states distributed as shown in Figure 2. The fixed overtaking opportunity road was generated using a single run of this Markov model for 100 km using the parameters detailed in the appendix. This lane will be referred to as the model with constant overtaking opportunities.
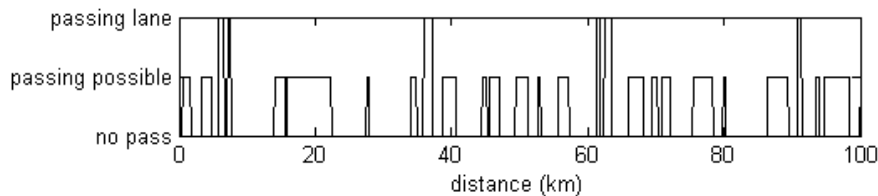
Figure 2: The model with constant overtaking opportunities

The lane with constant overtaking opportunities is distinct from the lanes used in the second phase which was conducted to explore the influence that variations in overtaking opportunities have on travel times. The state of the lane with the varying overtaking opportunities was determined during each simulation using a Markov model that is described in detail in the appendix.

Safe overtaking distance model

The safe overtaking distance is the distance from the modelled car to the furthest car that is travelling in the opposite direction that would impede an overtaking manoeuvre from occurring if it was going to be attempted. An overtaking manoeuvre would only be attempted if it were in a lane where overtaking was possible. Cars travelling in the opposite direction to the modelled vehicle enter this safe overtaking distance with a Poisson distribution. Cars will sequentially leave this safe overtaking distance at a rate which uses the distribution of speeds used in each simulation. The safe distance considered is a function of the speed difference between the vehicle being followed and the desired speed of the modelled vehicle. A shorter overtaking length is required if the modelled vehicle is travelling much slower than its target speed.

Overtaking

Overtaking is possible in the model when either the modelled vehicle is in a passing lane or in a lane where passing is possible if safe. When an overtaking opportunity arises, and the modelled vehicle is travelling at less than its desired speed the modelled vehicle overtakes and resumes its desired speed. If the modelled vehicle is following two or more slower vehicles, the modelled vehicle overtakes all slower vehicles when an overtaking opportunity arises.

For simplicity the distance required to move in front of the vehicle is set to be 40 metres in the model. This value is constant in the model, and hence the model considers all slower moving vehicles to be equally difficult to pass.

Simulations

Simulation was performed using Matlab (version 2009a) and processed using a desktop PC. Using the data available in table 1 approximately 1100 – 1800 vehicles are measured per day travelling in either direction at each site. A total of 1000 simulations were completed for each site for average daily traffic volumes of 1000, 3000 and 6000 vehicles per day averaged equally throughout the day. The density of the traffic is referred to as low, medium and high traffic density in this paper.

Results

Results of a single simulation

The results of a single typical simulation of the lane with constant overtaking opportunities described in Figure 2 are shown in Figure 3. For clarity, only the final 20 km of the simulation is shown. This figure shows how the model simulates travelling speed over the distance of the road. Slower vehicles are encountered, and an overtaking manoeuvre is only performed when the lane is an 'overtaking possible' lane and a clear safe overtaking distance is available (for example at 87 km) or the lane is an overtaking lane (for example at 92 km). The safe overtaking distance required to be empty is much less when trying

to overtake a slow moving vehicle than a faster moving vehicle, as a result the safe overtaking distance maximum is smaller when following a slower moving vehicle than behind a faster moving vehicle. This is shown in the figure with lower peak values of safe overtaking distance when the vehicle is travelling slower than when the vehicle is travelling faster.
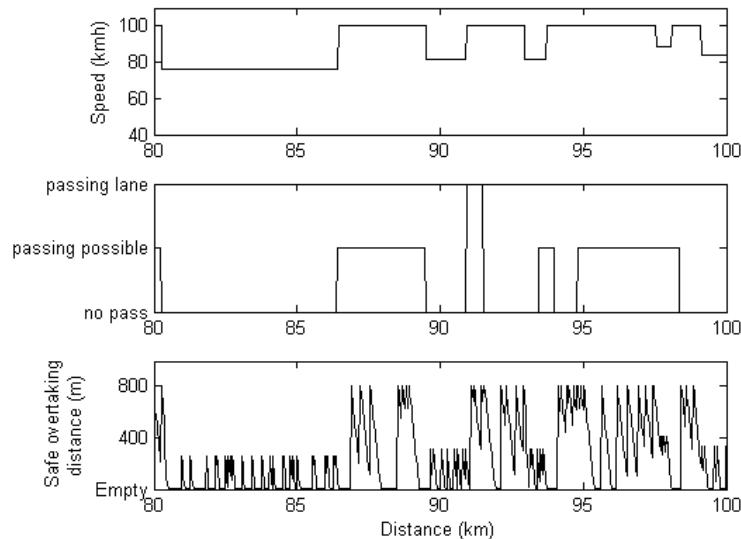

Figure 3: Typical simulation of the defined lane. Only the final 20 km of simulation is shown for clarity. High density traffic. Murray Bridge to Jervois speed distribution. 100 km/h desired speed.

Results of simulations performed with fixed overtaking opportunities (as defined by lane state)

After completion of the simulations, the variation in travel times is shown in Figure 4. This demonstrates that nearly all percentiles of journeys will see an increase in travel time. However, the increase in travel time will be less than the increase in travel time that would be expected if the modelled vehicle could travel at its desired speed for the entire journey. Using Equation (1), the theoretical increase in travel time due to the speed limit change from 110 km/h to 100 km/h is 10%. All of the simulated results suggest that the increase in travel time, while considering other traffic on the road, is less than this theoretical value. Percentage increases in travel time of between 0%, say when much of the journey is spent following a slow moving caravan, and 10%, when no slower moving vehicles are encountered, are possible using the model. 50% of all trips would expect an increase in travel time of between 6% and 10% depending on the density of traffic after a decrease in speed limit from 110 km/h to 100 km/h, provided the driver drives at the applicable speed limit as often as possible.

Furthermore, the number of overtaking manoeuvres performed can also be measured using the model. The number of overtaking manoeuvres performed by a vehicle is shown in Table 2. More overtaking is required at the higher speed limit than the lower speed limit.

The proportion of time spent travelling less than the desired speed and the proportion of time spent travelling more than 10 km/h less than the desired speed is presented for the sum of all journeys simulated is presented in Table 3. Higher proportions of time spent travelling at slower than desired speeds are observed when the speed limit is higher. Typically, the proportion of time spent travelling behind slower moving traffic approximately halves after the speed limit is dropped from 110 km/h to 100 km/h.

Results of simulations performed with varying overtaking opportunities (as defined by lane state)

The outcomes of simulations performed on roads with varying overtaking opportunities are shown in Figure 5, Table 4 and Table 5. Only slight differences are observed between the two sets of results. In the second case where the lane has varying overtaking opportunities, less overtaking manoeuvres are required for some journeys than for the fixed overtaking opportunities case. In the fixed overtaking opportunities

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case approximately 1% to 2% more time travelling behind slower moving vehicles is observed. These results are specific to the particular fixed overtaking opportunity lane that was selected.
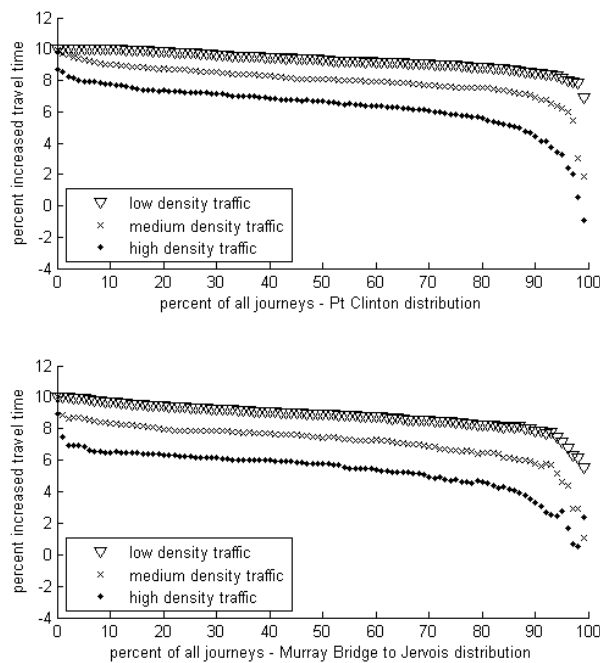

Figure 4: Percent increase in travel time for percentile of all journeys using the constant lane for percentile of all journeys completed before speed limit change for various traffic densities, Pt Clinton (above), Murray Bridge to Jervois (below).

Table 2: Number of overtaking manoeuvres performed to maintain travel speed for various percentiles of journeys using the constant overtaking opportunity lane.

Site		Low density traffic			Medium density traffic			High density traffic		
	Percentile	*15%*	*50%*	*85%*	*15%*	*50%*	*85%*	*15%*	*50%*	*85%*
Pt Clinton	110 km/h limit	0	1	2	4	5	6	9	11	13
	100 km/h limit	0	0	1	1	2	3	4	6	7
Murray Bridge - Jervois	110 km/h limit	1	2	3	6	7	9	13	15	16
	100 km/h limit	0	1	2	3	4	5	8	10	11

Table 3: Percent of time spent travelling at a speed less than desired speed, s_o, and more than 10 km/h slower than desired speed using the constant overtaking opportunity lane.

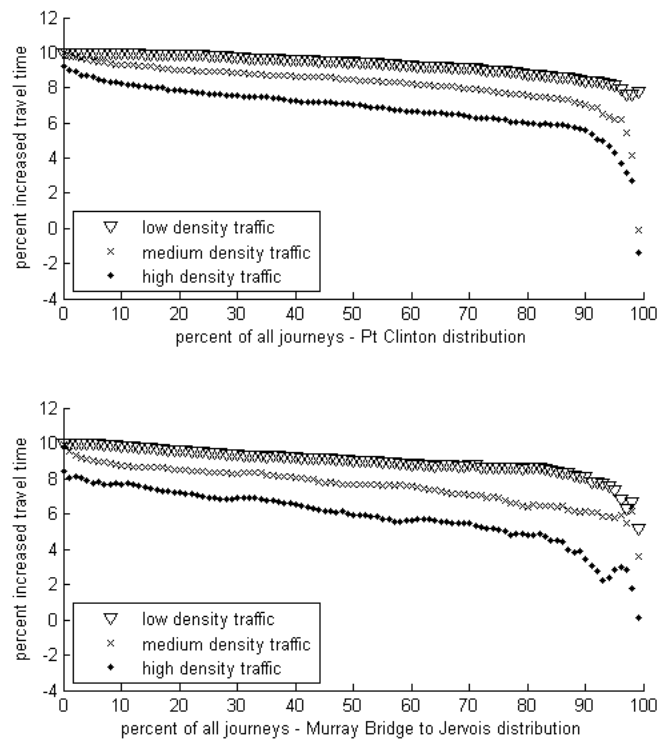| Site		Low density traffic		Medium density traffic		High density traffic	
	Speed difference	$< s_0$	$< s_0 - 10$	$< s_0$	$< s_0 - 10$	$< s_0$	$< s_0 - 10$
Pt Clinton	110 km/h limit	5.1	4.2	13.7	11.8	25.6	21.7
	100 km/h limit	2.2	1.6	6.6	4.2	14.1	9.0
Murray Bridge - Jervois	110 km/h limit	7.7	7.1	20.2	18.9	33.8	31.5
	100 km/h limit	4.3	3.5	12.7	10.6	23.1	19.1

Figure 5: Percentage increase in travel time after speed limit change using the variable lanes for percentile of all journeys completed before speed limit change for various traffic densities, Pt Clinton (above), Murray Bridge to Jervois (below).

Table 4: Number of overtaking manoeuvres performed to maintain travel speed for various percentiles of journeys using the variable overtaking opportunity lanes.

Site		Low density traffic			Medium density traffic			High density traffic		
	Percentile	*15%*	*50%*	*85%*	*15%*	*50%*	*85%*	*15%*	*50%*	*85%*
Pt Clinton	110 km/h limit	0	1	2	3	5	6	9	11	12
	100 km/h limit	0	0	1	1	2	3	4	5	7
Murray Bridge - Jervois	110 km/h limit	1	2	3	5	7	8	12	14	16
	100 km/h limit	0	1	1	3	4	5	7	9	10

Table 5: Percent of time spent travelling at a speed less than desired speed, s_o, and more than 10 km/h slower than desired speed using the variable overtaking opportunity lanes.

Site		Low density traffic		Medium density traffic		High density traffic	
	Speed difference	$< s_0$	$< s_0 - 10$	$< s_0$	$< s_0 - 10$	$< s_0$	$< s_0 - 10$
Pt Clinton	110 km/h limit	4.6	3.9	12.8	10.9	22.6	19.4
	100 km/h limit	2.1	1.3	6.5	4.3	13.0	8.2
Murray Bridge - Jervois	110 km/h limit	7.2	6.7	17.8	16.6	29.5	27.5
	100 km/h limit	4.2	3.5	11.4	9.4	20.6	17.1

Discussion

A simple Markov model has been developed to simulate the effect of other vehicles on the road network and their influence on travel time. The Markov model described includes provision to include real world speed distribution data that had been collected before and after a change in speed limit. Real world data that had been collected when a speed limit was changed in South Australia from 110 km/h to 100 km/h was applied to the model and results were simulated. The travel times, number of overtaking manoeuvres and proportion of time spent travelling at a speed less than the desired speed of the fastest law abiding driver were retrieved from the model. Three key points are found after evaluating the model for the fastest legal driver driving on the roads described in this paper with the distributions of vehicle speeds also described in the paper. The three key points relate to the increase in travel time, the number of overtaking manoeuvres required to maintain the drivers desired speed and the percentage of time spent behind vehicles travelling at a slower speed than the desired speed. Each key point is discussed in the following paragraphs.

Increases in travel time of between 2% and 8% are noted for high density traffic of 6000 vehicles per day on a road with fixed overtaking opportunities and a road with varying overtaking opportunities after a decrease in the speed limit. Increases in travel time of between 4% and 10% are noted for low density traffic of 1000 vehicles per day on both of these types of road. This compares with a theoretical increase in travel time of 10% if the speed limit is the sole consideration. Clearly, a decrease in the speed limit will see an increase in travel time if the road travelled is similar to the road modelled in this study, however, the increase in travel time is not as large as is suggested by the decrease in speed limit alone. Interestingly, this result is applicable to drivers who take every opportunity to overtake as is described in the model. For real drivers, however, the percentage increase in travel time will be less than is predicted in this model. This is due to the real driver possibly choosing not to overtake a vehicle that is going at a slightly slower speed than the desired speed.

The second key point is that the number of overtaking manoeuvres that were performed in order for the modelled driver to maintain their desired speed was greater when the desired speed was higher. This was true for both the road with the fixed amount of overtaking opportunities and the road with varying amounts of overtaking opportunities.

The third key point regards the percentage of time spent travelling below the desired speed of the vehicle. The amount of time that is spent travelling behind a slower moving vehicle is reduced when the lower speed limit is used.

An explanation as to why the number of overtaking manoeuvres and percentage of time spent travelling behind a slower moving vehicle is higher at higher speed limits can be found after analysis of the distribution of vehicle speeds shown in Figure 1. As is shown well at the Pt Clinton site, vehicles travelling at greater than 100 km/h tend to slow down toward a speed of 100 km/h after the speed limit change, while vehicles travelling less than 100 km/h tend maintain their speed. Vehicles travelling near the desired speed of the modelled vehicle are rarely caught by the modelled vehicle because their difference in speed is small. Thus, with a greater proportion of vehicles with a speed near the desired speed of the modelled vehicle the time spent following these vehicles is lower. In order to pass a vehicle, the modelled vehicle must firstly be following a slower vehicle. Because the incidence of following a slower moving vehicle is less, the need for an opportunity to overtake is also reduced. Clearly, the time spent following another vehicle, and the number of overtaking manoeuvres performed is related to the distribution of speeds.

This paper is aimed at addressing the concerns of a driver who claims that he or she never travels at a speed higher than the speed limit, nor does he or she travel below the speed limit unless following a slower vehicle. Such a driver might claim that travel time will increase 10% as a result of a speed limit change from 110 km/h to 100 km/h. This study has shown that this theoretical value is the upper bound of the increase in travel time for such a driver. A more realistic estimation of increase in travel time of between 4% and 10% is appropriate for such a driver when the influence of other traffic on the road is also considered. Additionally, this driver will spend less time following a slower vehicle, and be required

to overtake less often if the speed limit is lowered as described. An increase of travel time of between 4% and 10% equates to an increase of between 2.2 minutes and 5.5 minutes over a 100 km journey. This increase in travel time is balanced against approximately a 50% reduction in time spent following a slower vehicle and a lowering in the number of overtaking manoeuvres that are performed. The claim of the descried driver can be engaged and discussed using the results of this study.

The difference between the fixed overtaking opportunities case and the varying overtaking opportunities case is small. The lane used for the fixed overtaking opportunities case was generated using the Markov model described in the appendix. The negligible change in results indicates that a specific road may be modelled by a general road with the same average properties.

The model, like all models, has limitations. An assumption in the model is the Poisson distribution of vehicles on the road network. Platoons of cars are not modelled using a Poisson distribution and are hence neglected in the model. Dramatic changes in speed are modelled rather than a more steady acceleration or deceleration when a speed change occurs. Vehicles with difference characteristics for overtaking are not considered; such as the difference between overtaking a heavy articulated truck and a small passenger vehicle. Furthermore, overtaking of only one vehicle is considered equally difficult as passing a platoon of vehicles. Increases in travel time due to the speed reductions caused by geometrical constraints of the road are also not modelled. Each of the assumptions were made to simplify the model; in order to make the mathematics easier to apply.

Further improvements of the model include a more appropriate spatial distribution of vehicles on the road. Other cars, while for short periods of time can be considered to leave their origin with a Poisson distribution, over time these cars interact with each other. A lack of platooning of traffic is an acknowledged deficiency in this model, and as such, the results of the model can only be considered sensible for low volumes of traffic. Often, however, roads for which a speed reduction could potentially be considered are low traffic volume roads, for which a model like this becomes appropriate.

An interesting extension of the model would be to describe a distribution of travel times for the vehicles on the road that have a desired travelling speed given by the distribution of free speeds on the road. This value could then be used in economic evaluations of costs associated with a decreased speed limit, because the 50th percentile travel time of the average journey could be calculated. This will be included in future work with this model.

There are other future tasks for the model. The model needs to be tested in a real world situation. The model has been designed to take an input of an actual road lane state, ie. overtaking is possible if safe, overtaking not possible, and overtaking lanes if such measurements are available. The lane states of a real road will be measured and used in the simulation. The results of the simulation with real lane states will be compared travel times, with traffic volume counts being done simultaneously. The results of this real world testing can then be compared to the output of the model. Ultimately, it would be ideal to link the results of this study with the expected change in crash statistics.

Conclusions

The model presented in this paper is a simple tool for simulating the change in travel time after a change in speed limit is introduced. The simulation includes real data of distributions of travel speed. The results of the simulation can be used to engage the public during a debate on the lowering of speed limits including a less than expected reduction in travel time, a lowering of the number of overtaking manoeuvres that are required and reduction time spent travelling behind vehicles travelling at a slower speed than the speed limit. The model can be further refined to include additional parameters that will be important in the evaluation of total increase in travel time due to a speed limit reduction.

Acknowledgements

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Appendix

Markov models were used to simulate the vehicles on the road, and also the state of the lane when a lane with varying overtaking opportunities was used. This appendix describes a Markov model, and each of the models.

Model of vehicle speed

At each time step the modelled vehicle is assumed to have a speed given by a whole number of kilometres per hour. The set of speeds that include all speeds that are possible in the model are given by the following.

Let S(t) be the speed of the modelled vehicle at time step t. Then:

$$S(t) = s \, , s \in \{0 \, , 1 \, , 2, \dots, \text{max speed}\} \tag{A1}$$

The maximum speed is the fastest speed encountered in the model. Typically this speed is the fastest speed found in the distribution of other vehicles. However, this speed must be at least the desired speed of the modelled vehicle, s_0.

Transition from one state to another is only possible if the modelled vehicle encounters a vehicle that is travelling slower than the modelled vehicle, or, an overtaking manoeuvre is performed.

We define some other parameters necessary for this model.

Let
s_0:	Desired speed of modelled vehicle
S_i:	Value of speed - i km/h
Δt:	Time step
V_i:	Proportion of vehicles at speed S_i
$\overline{V}$:	Average speed of other vehicles
C_W:	Concentration of vehicles travelling in same direction as modelled vehicle (vehicles / km)

Then:

$$P\{S(t + \Delta t) = j | S(t) = i\}$$
$$= \begin{cases} (S_i - S_j) \cdot \Delta t \cdot C_W \cdot V_j & \text{where } s_0 \geq S_i > S_j \\ 1 - \sum_{S_i > S_k} (S_i - S_k) \cdot \Delta t \cdot C_W \cdot V_k & \text{where } s_0 \geq S_i = S_j \\ 0 & \text{otherwise} \end{cases} \quad (A2)$$

For instance, for the modelled vehicle, travelling at speed $S(i)$ to catch another vehicle travelling at speed $S(j)$, the modelled vehicle must be travelling faster than the vehicle it catches. The probability that it catches the slower vehicle is equal to the difference in speeds multiplied by the time step multiplied by the probability that a vehicle travelling at that particular speed is next one that is caught by the modelled vehicle. Overtaking is not considered at this stage of the model as it is dependent on both the state of the lane being traversed and the vehicles travelling in the opposite direction to the modelled direction.

Reaching a platoon of traffic was not considered in the model. The model assumes that there are sufficiently low number of cars, and sufficiently high amounts of overtaking opportunities that the effect of platoons of traffic can be neglected.

Modelling of cars in immediate safe overtaking zone travelling the opposite direction

Overtaking can only be performed on a rural road in South Australia that does not have overtaking lanes if the lane markings indicate that overtaking is possible, given by a dashed centre line, and there is sufficient clear space in the oncoming lane for an overtaking manoeuvre to be safely performed. The traffic in the opposing lane was modelled using a Markov model, which modelled the distance to the furthest car that could be considered to impede an overtaking manoeuvre.

Let H be the distance, in metres, to the furthest car that would be considered before making a decision to overtake.

$$H(S(t), t) = h \in \{0, 10, 20, \dots \max(H)\} \quad (A3)$$

The maximum value of H, while theoretically unbounded, is chosen for computational simplicity. A distance of 800 m was used in all simulations in this study. A schematic for this value is shown in Figure A1.

We also define some other parameters necessary for the formulation of this model.

Let

OTD: Overtaking distance: distance required to move ahead of vehicle in front to perform an overtaking manoeuvre

C_A: Concentration of vehicles travelling in opposite direction as modelled vehicle (vehicles / km)

B: Additional speed that the modelled car is willing to travel over S_0 to perform an overtake

In a real world scenario, when a vehicle is travelling behind a slower vehicle, an overtaking manoeuvre will not be considered if there exists a vehicle travelling in the lane carrying traffic in the opposite direction if within some distance: H_{max}. A schematic for this distance is shown in Figure A1 and this distance is calculated in the following way:

$$H_{max}(S(t), t) = \frac{OTD}{S_0 - S(t) + B}(S_0 + B + \overline{V}) \tag{A4}$$

We assume that the cars travelling in the opposite direction that arrive in the zone bounded by $H = 0$ and $H = H_{max}$ can be modelled by a Poisson arrival process. Therefore the probability of a car coming within the considered safe overtaking zone is independent of any other car entering the safe overtaking zone. An assumption is made that the effect of a platoon of traffic arriving from the other direction can be neglected.
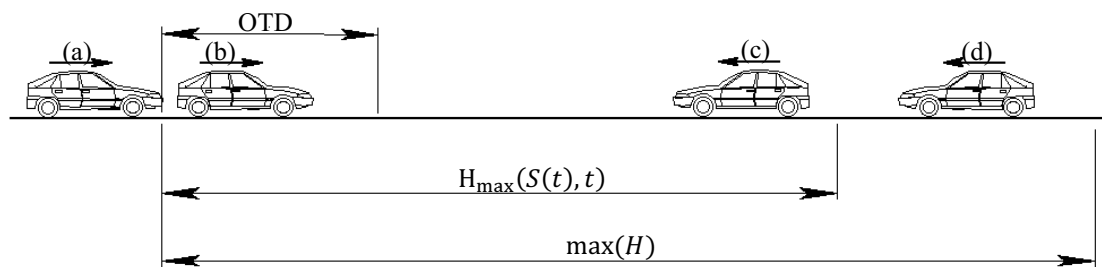

Figure A1: Schematic of overtaking zone parameters. Vehicle (a) is the modelled vehicle, following a slower moving vehicle (b), both at speed $S(t) < S_0$. The oncoming vehicle (c) is modelled as an oncoming vehicle as it is inside the distance $H_{max}(S(t), t)$. Vehicle (d) is not modelled until it is inside H_{max}. H_{max} is a function of travelling speed of vehicle (a) and is constrained to have a maximum value of max*(H)*.

The probability of a car entering the safe overtaking zone space for each travelling speed, and hence of the safe overtaking zone becoming in time Δt is:

$$P\{H((S(t), t + \Delta t) = H_{max}|H(S(t), t) = i\} = (S(t) + \overline{V}) \cdot \Delta t \cdot C_A \tag{A5}$$

This probability holds regardless of the amount of the safe overtaking zone already consumed. This allows more than one vehicle to occupy the safe overtaking zone while only considering the vehicle which will affect an overtaking manoeuvre for the longest time period which is the one furthest from the modelled vehicle, but still in the safe overtaking zone.

Once a vehicle is in the safe overtaking zone it proceeds out of the zone. At each time step, Δt, the vehicle will have moved a distance equal to the sum of the speed that the car in the safe overtaking zone is travelling and the modelled vehicle multiplied by the time step. For instance, if the modelled car is travelling at 100 km/h and a car in the safe overtaking zone is travelling at 60 km/h, for a time step of 1 second, the car in the safe overtaking zone will move 44.4 m toward the end of the safe overtaking zone. 44.4 m is not within the set of states that can be occupied by $H(s(t), t)$, however, so the value is rounded to the nearest value in the set h, ie. 40 m. The vehicles in the safe overtaking zone have a distribution of

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speeds given by the distribution of speeds, V_k, allocated to the model. Therefore the distance moved by any car out of the safe overtaking zone is distributed according to the distribution of speeds at each time step. The change in safe overtaking zone distance will not generally be equivalent to one of the states available in the safe overtaking zone model, h. The change in safe overtaking zone distance is set to the nearest state in h to overcome this problem.

Therefore, in order for the safe overtaking zone to un-fill:

$$P\{H(S(t), t + \Delta t) = j | H(S(t), t) = i\} = \begin{cases} \sum_{G_k \in h = i - j} V_k & i > j, j > 0 \\ \sum_{G_k \in h > i - j} V_k & i > j, j \leq 0 \\ 0 & otherwise \end{cases} \quad \text{(A6)}$$

Where

$$G_k = (S(t) + S_k) \cdot \Delta t, G_k \in h \quad \text{(A7)}$$

Lane model

The state of the lane for the phase where varying overtaking opportunities arise is also described using a Markov model. The distribution of these states could be directly modelled from a real road without the need for Markov transition probabilities. However, in the absence of detailed measurements of the lane state on a long stretch of road, the state of the lane modelled using a stochastic process. The fixed overtaking opportunity road was generated using a single run of this Markov model for 100 km.

Let the states be described by L:

$$L(S(t), t) = l \in \{1, 2, 3\} \quad \text{(A8)}$$

Where the passing lane states are:

$$L(S(t), t) = \begin{cases} 1 & \text{where no overtaking is allowed, eg. on bends} \\ 2 & \text{where overtaking is allowed if safe} \\ 3 & \text{where an overtaking lane exists} \end{cases} \quad \text{(A9)}$$

The following parameters are needed to fully describe the transition matrix.

Let
D_i: Average Length of Lane in state i
$Le_{i,j}$:   Probability of moving to state j if the previous state was i, $i \neq j$

The average length of each of the lane states D_i can be estimated. The values of these D_i can be used to determine the probability that the current lane ends within Δt, and therefore if it will continue for another time step:

$$P\{L(S(t), t + \Delta t) = i | L(S(t), t) = i\} = 1 - \frac{S(t) \cdot \Delta t}{D_i} \quad \text{(A10)}$$

The probability of transitioning to a state that is different in a single time step is given by the following equation:

$$P\{L(S(t), t + \Delta t) = i | L(S(t), t) = j\} = \frac{S(t) \cdot \Delta t}{D_i} \cdot Le_{i,j} \quad \text{(A11)}$$

In particular, some values needed to be set for this paper. The average distances for each lane type used in the lane state model were:

$$D_1 = 2\ km$$
$$D_2 = 2\ km$$
$$D_3 = 1\ km$$

And the transitions between each state in the lane type model were:

$$Le_{1,2} = 0.8\ ,\quad Le_{1,3} = 0.2$$
$$Le_{2,1} = 1.0,\quad Le_{2,3} = 0.0$$
$$Le_{3,1} = 1.0,\quad Le_{3,2} = 0.0$$

Using these transitions, the lane will only transition to a passing lane from a 'no overtaking' lane and not from a 'overtaking possible' lane and the only state that can follow either a 'passing lane' or a 'passing possible' lane is a 'no passing possible' state.

Using these known transitions between states, it is possible to determine the average time spent in each state. The average distance spent in the lane where no passing is allowed is 52.63%, the average distance in a passing is possible lane is 42.11% and the average distance in a passing lane is 5.26%. The transitions, $Le_{i,j}$ and D_i, could be changed to adjust these average lane states if desired.

Overtaking

Overtaking is allowed by considering all three models together. Overtaking will occur if one of the following two conditions are met in conjunction with $S(t) < S_0$:

1. The modelled vehicle is in a passing lane
2. The modelled vehicle is in a passing-possible lane and the safe overtaking zone of the modelled vehicle is clear.

If either of these conditions is satisfied then the modelled car resumes its desired speed, S_0 and maintains its current lane state and safe overtaking zone state until such time as they change. No time is considered for the acceleration required to reach maximum speed. A safety margin for overtaking can be built into the safe overtaking zone model, through the distance, *OTD*, required to pass around a slower moving vehicle.

Time step

The time step chosen for the model needs to be sufficiently small such that two events have a negligible opportunity to occur in one time step. Examples of two events occurring simultaneously include two vehicles filling the safe overtaking zone in one time step, or the lane state changing from passing possible, to passing not possible to a passing lane.

Parameters used in the simulations

Particular parameters were used throughout this paper during the simulations. These are itemised here:

$$\Delta t = 1\ second$$
$$OTD = 0.04\ km$$
$$B = 10\ kph$$